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10MAT41

Fourth Semester B.E. Degree Examination, December 2012

Engineering Mathematics – IV

Time: 3 hrs.

Max. Marks:100

Note: Answer FIVE full questions, selecting at least TWO questions from each part.

PART – A

- 1 a. Using the Taylor's series method, solve the initial value problem $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ at the point $x = 0.1$ (06 Marks)
- b. Employ the fourth order Runge-Kutta method to solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$ at the points $x = 0.2$ and $x = 0.4$. Take $h = 0.2$. (07 Marks)
- c. Given $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$, $y(0.1) = 1.1169$, $y(0.2) = 1.2773$, $y(0.3) = 1.5049$. Find $y(0.4)$ using the Milne's predictor-corrector method. Apply the corrector formula twice. (07 Marks)
- 2 a. Employing the Picard's method, obtain the second order approximate solution of the following problem at $x = 0.2$.
 $\frac{dy}{dx} = x + yz$, $\frac{dz}{dx} = y + zx$, $y(0) = 1$, $z(0) = -1$. (06 Marks)
- b. Using the Runge-Kutta method, find the solution at $x = 0.1$ of the differential equation $\frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} - 2xy = 1$ under the conditions $y(0) = 1$, $y'(0) = 0$. Take step length $h = 0.1$. (07 Marks)
- c. Using the Milne's method, obtain an approximate solution at the point $x = 0.4$ of the problem $\frac{d^2y}{dx^2} + 3x \frac{dy}{dx} - 6y = 0$, $y(0) = 1$, $y'(0) = 0.1$. Given that $y(0.1) = 1.03995$, $y(0.2) = 1.138036$, $y(0.3) = 1.29865$, $y'(0.1) = 0.6955$, $y'(0.2) = 1.258$, $y'(0.3) = 1.873$. (07 Marks)
- 3 a. If $f(z) = u + iv$ is an analytic function, then prove that $\left(\frac{\partial}{\partial x} |f(z)|\right)^2 + \left(\frac{\partial}{\partial y} |f(z)|\right)^2 = |f'(z)|^2$. (06 Marks)
- b. Find an analytic function whose imaginary part is $v = e^x \{(x^2 - y^2) \cos y - 2xy \sin y\}$. (07 Marks)
- c. If $f(z) = u(r, \theta) + iv(r, \theta)$ is an analytic function, show that u and v satisfy the equation $\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} = 0$. (07 Marks)
- 4 a. Find the bilinear transformation that maps the points $1, i, -1$ onto the points $i, 0, -i$ respectively. (06 Marks)
- b. Discuss the transformation $W = e^z$. (07 Marks)
- c. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$, where C is the circle $|z| = 3$. (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
 2. Any revealing of identification, appeal to evaluator and/or equations written eg. 42+8 = 50, will be treated as malpractice.

PART – B

- 5 a. Express the polynomial $2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. (06 Marks)
- b. Obtain the series solution of Bessel's differential equation $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - n^2)y = 0$ in the form $y = AJ_n(x) + BJ_{-n}(x)$. (07 Marks)
- c. Derive Rodrique's formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$. (07 Marks)

- 6 a. State the axioms of probability. For any two events A and B, prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. (06 Marks)
- b. A bag contains 10 white balls and 3 red balls while another bag contains 3 white balls and 5 red balls. Two balls are drawn at random from the first bag and put in the second bag and then a ball is drawn at random from the second bag. What is the probability that it is a white ball? (07 Marks)
- c. In a bolt factory there are four machines A, B, C, D manufacturing respectively 20%, 15%, 25% 40% of the total production. Out of these 5%, 4%, 3% and 2% respectively are defective. A bolt is drawn at random from the production and is found to be defective. Find the probability that it was manufactured by A or D. (07 Marks)

- 7 a. The probability distribution of a finite random variable X is given by the following table:

x_i	-2	-1	0	1	2	3
$p(x_i)$	0.1	k	0.2	2k	0.3	k

- Determine the value of k and find the mean, variance and standard deviation. (06 Marks)
- b. The probability that a pen manufactured by a company will be defective is 0.1. If 12 such pens are selected, find the probability that (i) exactly 2 will be defective, (ii) at least 2 will be defective, (iii) none will be defective. (07 Marks)
- c. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation, given that $A(0.5) = 0.19$ and $A(1.4) = 0.42$, where $A(z)$ is the area under the standard normal curve from 0 to $z > 0$. (07 Marks)
- 8 a. A biased coin is tossed 500 times and head turns up 120 times. Find the 95% confidence limits for the proportion of heads turning up in infinitely many tosses. (Given that $z_c = 1.96$) (06 Marks)
- b. A certain stimulus administered to each of 12 patients resulted in the following change in blood pressure:
5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4 (in appropriate unit)
Can it be concluded that, on the whole, the stimulus will change the blood pressure. Use $t_{0.05}(11) = 2.201$. (07 Marks)
- c. A die is thrown 60 times and the frequency distribution for the number appearing on the face x is given by the following table:

x	1	2	3	4	5	6
Frequency	15	6	4	7	11	17

Test the hypothesis that the die is unbiased.

(Given that $\chi_{0.05}^2(5) = 11.07$ and $\chi_{0.01}^2(5) = 15.09$)

(07 Marks)

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Fourth Semester B.E. Degree Examination, December 2012

Microcontrollers

Time: 3 hrs.

Max. Marks:100

Note: Answer FIVE full questions, selecting at least TWO questions from each part.

PART – A

- 1 a. Mention the difference between RISC and CISC processor. (06 Marks)
- b. Explain function of following pins of 8051:
- \overline{EA}
 - \overline{ALE}
 - \overline{PSEN}
 - RST
- c. Explain the internal memory organization of 8051. (06 Marks)
- 2 a. With necessary examples, explain immediate, bit direct and indexed addressing mode of 8051. (06 Marks)
- b. Explain the effect of following instructions:
- MOVX A, @dptr
 - DJNZ R3, next
 - DA A
 - LJMP label
- c. Write a program segment to realize following:
- Exchange contents of external data memory 8100 h with contents of internal data memory 40 h.
 - Exchange contents of A-register and B-register using stack. (06 Marks)
- 3 a. What do you understand by assembler directives? Explain the following assembler directives:
- ORG
 - END
 - EQU
- b. Write an ALP to convert a 2-digit BCD number to binary. (06 Marks)
- c. Write a delay program to generate a delay of 10 ms. Assume a crystal of 11.0592 MHz. Show delay calculation clearly. (06 Marks)
- 4 a. Explain the usage of the port pins of 8051. (06 Marks)
- b. With necessary interface diagram, write a C program to generate a triangular wave using DAC interface. (06 Marks)
- c. With necessary interface diagram, write a program to display "VTU2012" on a LCD interface. (08 Marks)

PART – B

- 5 a. Mention the difference between counter mode and timer mode of operation. With necessary format, explain the various bits of TMOD-SFR. (06 Marks)
- b. Write an 8051 C program to generate a square wave of 2 kHz using timer 1, mode 2. Show the calculations clearly. Assume a crystal frequency of 11.0592 MHz. (08 Marks)
- c. Explain the interrupts of 8051 clearly mentioning the vector address and priorities. (06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
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- 6 a. Explain the various modes of serial communication operation. (06 Marks)
b. Write a program to transmit a message "VTU" serially at a band rate of 9600. Take crystal frequency as 11.0592 MHz. (08 Marks)
c. Explain with a neat diagram, the functional block diagram of 8255. (06 Marks)
- 7 a. With necessary block diagram, explain the architecture of MSP 430. (08 Marks)
b. Explain clock system of MSP 430. (08 Marks)
c. Explain the use of registers P_XDIR and P_XOUT. (04 Marks)
- 8 a. Explain various low power operating modes of MSP 430. (10 Marks)
b. Explain the bits of TCON register. Write an 8051 C program to toggle only bit P1.5 continuously every 50 msec. Use timer 1 to generate the delay. Assume XTAL = 11.0592 MHz. (10 Marks)

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Fourth Semester B.E. Degree Examination, December 2012
Control Systems

Time: 3 hrs.

Max. Marks:100

Note: Answer FIVE full questions, selecting at least TWO questions from each part.

PART – A

- 1 a. Compare linear and non-linear control system. (04 Marks)
 b. For the two port network shown in Fig. Q1 (b), obtain the transfer functions, i) $\frac{V_2(s)}{V_1(s)}$ and ii) $\frac{V_1(s)}{I_1(s)}$ (08 Marks)

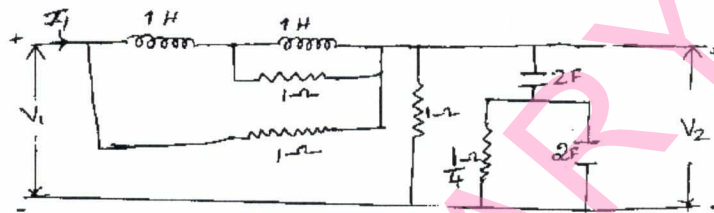


Fig. Q1 (b)

- c. For the rotational system shown in Fig. Q1 (c), i) Draw the mechanical network ii) Write the differential equations iii) Obtain torque to voltage analogy. (08 Marks)

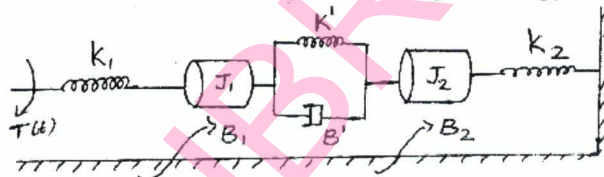


Fig. Q1 (c)

- 2 a. Illustrate how to perform the following, in connection with block diagram reduction rules:
 i) Shifting a take-off point after a summing point.
 ii) Shifting a take-off point before a summing point. (04 Marks)
 b. The performance equations of a controlled system are given by the following set of linear algebraic equations:
 i) Draw the block diagram.

- ii) Find the overall transfer function $\frac{C(s)}{R(s)}$ using block diagram reduction technique.
 $E_1(s) = R(s) - H_3(s)C(s)$; $E_2(s) = E_1(s) - H_1(s)E_4(s)$; $E_3(s) = G_1(s)E_2(s) - H_2(s)C(s)$
 $E_4(s) = G_2(s)E_3(s)$; $C(s) = G_3(s)E_4(s)$ (08 Marks)

- c. Draw the corresponding signal flow graph for the given block diagram is shown in Fig. Q2 (c) and obtain the overall transfer function by Mason's gain formula. (08 Marks)

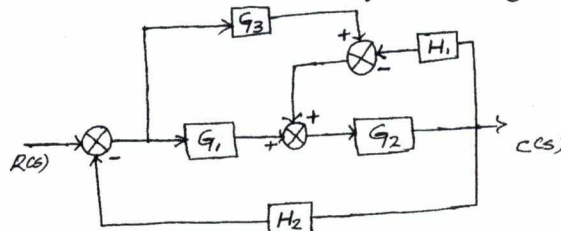


Fig. Q2 (c)

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- 3 a. Derive the expression for peak time. (04 Marks)
 b. The loop transfer function of a feed back control system is given by,

$$G(s)H(s) = \frac{100}{s^2(s+4)(s+12)}$$

- i) Determine the static error co-efficients.
 ii) Determine the steady state error for the input $r(t) = 2t^2 + 5t + 10$ (08 Marks)

- c. A system is given by differential equation, $\frac{d^2 y(t)}{dt^2} + 4 \frac{dy(t)}{dt} + 8y(t) = 8x(t)$.

Where $y(t)$ = output and $x(t)$ = input.

- Determine i) peak time ii) peak over shoot iii) settling time
 iv) expression of the output response. (08 Marks)

- 4 a. Define the term stability applied to control system and what is the difference between absolute stability and relative stability. (04 Marks)

- b. Using Routh's criterion determine the stability of following systems:

- i) Its open loop transfer function has poles at $s = 0, s = -1, s = -3$ and zero at $s = -5$. Gain $K = 10$.
 ii) It is a type one system with an error constant of 10 sec^{-1} and poles at $s = -3$ and $s = -6$ (08 Marks)

- c. Using RH criterion determine the stability of the system having the characteristic equation, $s^4 + 10s^3 + 36s^2 + 70s + 75 = 0$ has roots more negative than $s = -2$. (08 Marks)

PART - B

- 5 a. The open-loop transfer function of a feed back control system in $G(s)H(s) = \frac{K}{(s+1)(s+2)(s+3)}$, check whether the following points are on the root locus. If so, find the value of K at these points. i) $s = -1.5$ ii) $s = -0.5 + j2$ (06 Marks)

- b. Sketch the root locus plot for a negative feed back control system characterized by an open loop transfer function, $G(s)H(s) = \frac{K}{s(s+3)(s^2+3s+11.2s)}$. Comment on stability. (14 Marks)

- 6 a. State the advantages and limitations of frequency domain approach. (06 Marks)
 b. Determine the transfer function, of a system whose asymptotic gain plot is shown in Fig. Q6 (b). (10 Marks)

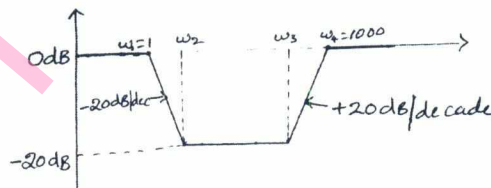


Fig. Q6 (b)

- c. List the effects of lead compensation. (04 Marks)

- 7 a. Draw polar plot of $G(s)H(s) = \frac{100}{(s+2)(s+4)(s+8)}$. (04 Marks)

- b. Explain Nyquist stability criterion. (06 Marks)

- c. Sketch the Nyquist plot for, $GH(s) = \frac{K}{s(s+1)(s+2)}$. Then, find the range of K for closed loop stability. (10 Marks)

- 8 a. Define the following terms: i) state ii) state variables iii) state space. (06 Marks)

- b. List the advantages of state variable analysis. (04 Marks)

- c. Obtain the state transition matrix for, $A = \begin{bmatrix} 0 & -1 \\ 2 & -3 \end{bmatrix}$. (10 Marks)

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Fourth Semester B.E. Degree Examination, December 2012
Signals and Systems

Time: 3 hrs.

Max. Marks:100

Note: Answer FIVE full questions, selecting at least TWO questions from each part.

PART – A

- 1 a. Determine whether the following systems are:
i) Memoryless, ii) Stable iii) Causal iv) Linear and v) Time-invariant.
- $y(n) = nx(n)$
 - $y(t) = e^{x(t)}$
- (10 Marks)**
- b. Distinguish between: i) Deterministic and random signals and
ii) Energy and periodic signals. **(06 Marks)**
- c. For any arbitrary signal $x(t)$ which is an even signal, show that $\int_{-\infty}^{\infty} x(t)dt = 2 \int_0^{+\infty} x(t)dt$. **(04 Marks)**
- 2 a. Find the convolution integral of $x(t)$ and $h(t)$, and sketch the convolved signal, $x(t) = (t-1)\{u(t-1) - u(t-3)\}$ and $h(t) = [u(t+1) - 2u(t-2)]$. **(12 Marks)**
- b. Determine the discrete-time convolution sum of the given sequences.
 $x(n) = \{1, 2, 3, 4\}$ and $h(n) = \{1, 5, 1\}$ **(08 Marks)**
- 3 a. Determine the condition of the impulse response of the system if system is,
i) Memory less ii) Stable. **(06 Marks)**
- b. Find the total response of the system given by,
 $\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 2x(t)$ with $y(0) = -1$; $\frac{dy(t)}{dt}\bigg|_{t=0} = 1$ and $x(t) = \cos(t)u(t)$. **(14 Marks)**
- 4 a. One period of the DTFS coefficients of a signal is given by, $x(k) = (\frac{1}{2})^k$, on $0 \leq k \leq 9$.
Find the time-domain signal $x(n)$ assuming $N = 10$. **(06 Marks)**
- b. Prove the following properties of DTFs: i) Convolution ii) Parseval relationship
iii) Duality iv) Symmetry. **(14 Marks)**

PART – B

- 5 a. Find the DTFT of the sequence $x(n) = \alpha^n u(n)$ and determine magnitude and phase spectrum. **(04 Marks)**
- b. Plot the magnitude and phase spectrum of $x(t) = e^{-at} u(t)$. **(08 Marks)**
- c. Find the inverse Fourier transform of the spectra, $x(j\omega) = \begin{cases} 2 \cos(\omega), & |\omega| < \pi \\ 0, & |\omega| > 0 \end{cases}$ **(08 Marks)**

- 6 a. Find the frequency response and impulse response of the system described by the differential equation.

$$\frac{d^2}{dt^2} y(t) + 5 \frac{d}{dt} y(t) + 6y(t) = -\frac{d}{dt} x(t) \quad (08 \text{ Marks})$$

- b. State sampling theorem. Explain sampling of continuous time signals with relevant expressions and figures. (06 Marks)
- c. Find the Nyquist rate for each of the following signals:
- i) $x_1(t) = \sin c(200t)$ ii) $x_2(t) = \sin c^2(500t)$ (06 Marks)

- 7 a. Prove the complex conjugation and time-advance properties. (06 Marks)

- b. Find the z-transform of the signal along with ROC.
 $x(n) = n \sin\left(\frac{\pi}{2} n\right) u(n)$ (06 Marks)

- c. Determine the inverse z-transform of the following $x(z)$ by partial fraction expansion method,

$$x(z) = \frac{z+2}{2z^2 - 7z + 3}$$

- if the ROCs are i) $|z| > 3$ ii) $|z| < \frac{1}{2}$ and iii) $\frac{1}{2} < |z| < 3$. (08 Marks)

- 8 a. A system has impulse response $h(n) = \left(\frac{1}{2}\right)^n u(n)$, determine the input to the system if the output is given by,

$$y(n) = \frac{1}{3} u(n) + \frac{2}{3} \left(-\frac{1}{2}\right)^n u(n). \quad (08 \text{ Marks})$$

- b. Solve the following difference equation using unilateral z-transform,

$$y(n) - \frac{3}{2} y(n-1) + \frac{1}{2} y(n-2) = x(n), \quad \text{for } n \geq 0, \quad \text{with initial conditions } y(-1) = 4,$$

$$y(-2) = 10, \quad \text{and } x(n) = \left(\frac{1}{4}\right)^n u(n). \quad (12 \text{ Marks})$$

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Fourth Semester B.E. Degree Examination, December 2012
Fundamentals of HDL

Time: 3 hrs.

Max. Marks:100

**Note: Answer FIVE full questions, selecting
at least TWO questions from each part.**

PART – A

- 1
 - a. Write switch level description in VHDL for the inverter circuit with nmos and pmos. Explain the advantages of this type description over the other types. (08 Marks)
 - b. Write the result of all shift and rotate operations in VHDL after applying them to a 7-bit vector A = 1001010 for one position. (06 Marks)
 - c. Explain verilog data types. (06 Marks)
- 2
 - a. Design 2×2 unsigned combinational array multiplier and write the VHDL code for the same. (08 Marks)
 - b. Draw the block diagram of a 3-bit carry-look ahead adder and write data flow description for its Boolean functions in verilog. (08 Marks)
 - c. Explain signal declaration and assignment with examples. (04 Marks)
- 3
 - a. Write the behavioural description for a half adder using verilog. (04 Marks)
 - b. Write the VHDL description for SR flip-flop using case statement and variable declaration. (06 Marks)
 - c. Explain Booth algorithm with flow chart. Write VHDL description to multiply two 4-bit numbers -5 and 7. (10 Marks)
- 4
 - a. Write gate level diagram and verilog structural description for D-latch. (08 Marks)
 - b. What is binding? Discuss binding between.
 - i) Entity and architecture. (08 Marks)
 - ii) Library and component. (04 Marks)
 - c. What are the advantages of HDL structural description? (04 Marks)

PART – B

- 5
 - a. Write a VHDL function to find the greater of two signed numbers. (06 Marks)
 - b. Write HDL description to convert signed binary to the integer using task. (08 Marks)
 - c. What is the significance of procedure, task and function? Differentiate between them. (06 Marks)

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- 6 a. Describe the development of HDL code for an arithmetic logic unit and write the verilog code for 16-bit ALU to perform 8 operations. (08 Marks)
- b. Write the block diagram and function table of a 3SRAM. Using this write a verilog description for 16×8 SRAM. (08 Marks)
- c. How to attach a package to the VHDL module? Explain with example. (04 Marks)
- 7 a. Write mixed language description of a JK flip-flop with clear, invoking VHDL entity from verilog module. (08 Marks)
- b. Describe full adder using two half adder invoking verilog module from VHDL entity. (08 Marks)
- c. Explain the necessity of mixed language description. (04 Marks)
- 8 a. What is synthesis? With a neat flow chart, explain the steps involved in a synthesis process. (06 Marks)
- b. Design gate level synthesis and write VHDL description for the information given below:

Inputs		Output
a	b	z
00	0 – 7	$z = b$
01	0 – 7	$z = b + 4$
10	0 – 7	$z = b/2$
11	x x	$z = 15$
x x	8 – 15	$z = 15$

(14 Marks)

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Fourth Semester B.E. Degree Examination, December 2012
Linear ICs and Applications

Time: 3 hrs.

Max. Marks:100

Note: 1. Answer FIVE full questions, selecting at least TWO questions from each part.
2. Missing data may be assumed.

PART – A

- 1 a. Explain the basic circuit of operational amplifier. (08 Marks)
 b. Define Slew rate and unity gain band width. What is the effect of slew rate on the output voltage of an op-amp? (06 Marks)
 c. Design an inverting amplifier using op-amp 741. The voltage gain is to be 50 and the output voltage amplitude is to be 2.5V. (06 Marks)
- 2 a. With a neat circuit diagram, explain capacitor coupled voltage follower with relevant design steps. (10 Marks)
 b. With a neat circuit diagram, explain the design of high input impedance capacitor coupled non-inverting amplifier. (10 Marks)
- 3 a. Explain how stability of high gain amplifier and lower gain amplifier is analyzed. (06 Marks)
 b. Explain phase-lag compensation and phase lead compensation. (10 Marks)
 c. Determine the upper cutoff frequency for a (i) voltage follower, (ii) unity gain inverting amplifier using a 741 op-amp. Given that UGB of 741 is 800 kHz. (04 Marks)
- 4 a. With a neat circuit diagram, explain op-amp as precision voltage source. (10 Marks)
 b. Briefly explain non-saturating precision half wave rectifier. (05 Marks)
 c. Using Bipolar op-amps with $V_{CC} = \pm 15V$, design the high input impedance precision full wave rectifier. The input peak voltage is to be 1V and no amplification is to occur. (05 Marks)

PART – B

- 5 a. A $\pm 5V$, 10 kHz square wave from a signal source with a resistance of 100Ω is to have its positive peak clamped precisely at ground level. Tilt on the output is not to exceed 1% of the peak amplitude of the wave. Design a suitable op-amp circuit for precision clamping using a supply of $\pm 12V$. (08 Marks)
 b. Briefly explain op-amp square wave/triangular wave generator with relevant circuit diagram, waveforms and expressions. (08 Marks)
 c. Using a BIFET op-amp with a supply of $\pm 12V$, design a Wein bridge oscillator to have an output frequency of 15 kHz. (04 Marks)
- 6 a. With a neat circuit diagram, waveform and expressions explain capacitor coupled non-inverting crossing detector. (08 Marks)
 b. Explain op-amp inverting Schmitt trigger for adjustable UTP and LTP. (08 Marks)
 c. Using a 741 op-amp design a first order active low pass filter to have a cut off frequency of 1 kHz. (04 Marks)
- 7 a. With a neat internal diagram of IC723, explain the functions of each block. (10 Marks)
 b. Explain basic switching regulator circuit with relevant expressions. Mention its advantages and disadvantages. (10 Marks)
- 8 a. Explain 555 timer as monostable multivibrator with relevant circuit diagram, waveforms and expressions. (07 Marks)
 b. Explain op-amp D/A converter with R and 2R resistors. (07 Marks)
 c. Draw the block diagram of PLL and explain it. (06 Marks)

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Fourth Semester B.E. Degree Examination, December 2012

Advanced Mathematics – II

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions.

- 1**
- Prove that the angle between two lines whose direction cosines are (l_1, m_1, n_1) and (l_2, m_2, n_2) is $\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$. (06 Marks)
 - Find the projection of the line AB on CD where $A = (1, 3, 5)$, $B = (6, 4, 3)$, $C = (2, -1, 4)$ and $D = (0, 1, 5)$. (07 Marks)
 - Find the angle between any two diagonals of cube. (07 Marks)
- 2**
- Find the equation of the plane passing through the points $(3, 1, 2)$ and $(3, 4, 4)$ and perpendicular to $5x + y + 4z = 0$. (06 Marks)
 - Show that the points $(0, -1, 0)$, $(2, 1, -1)$, $(1, 1, 1)$ and $(3, 3, 0)$ are coplanar. (07 Marks)
 - Find the equation of the plane through the points $(1, 0, -1)$, $(3, 2, 2)$ and parallel to the line $\frac{x-1}{1} = \frac{1-y}{2} = \frac{z-2}{3}$. (07 Marks)
- 3**
- Find the value of λ such that the vectors $\lambda i + j + 2k$, $2i - 3j + 4k$ and $i + 2j - k$ are coplanar. (06 Marks)
 - If $\vec{a} = 4i + 2j - k$, $\vec{b} = 2i - j$ and $\vec{c} = j - 3k$, find (i) $(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c})$, (ii) $(\vec{a} \times \vec{b}) \times (\vec{b} \times \vec{c})$. (07 Marks)
 - Find the cosine and sine of the angle between the vectors $2i - j + 3k$ and $i - 2j + 2k$. (07 Marks)
- 4**
- Find the components of velocity and acceleration at $t = 2$ on the curve, $\vec{r} = (t^2 + 1)i + (4t - 3)j + (2t^2 - 6t)k$ in the direction of $i + 2j + 2k$. (06 Marks)
 - Find the angle between the tangents to the curve $\vec{r} = \left\{ t - \frac{t^3}{3} \right\} i + t^2 j + \left\{ t + \frac{t^3}{3} \right\} k$ at $t = \pm 3$. (07 Marks)
 - Find the directional derivative of $\phi = x^2 yz + 4xz^2$ at $(1, -2, -1)$ along $2i - j - 2k$. (07 Marks)
- 5**
- If $\vec{F} = \nabla(xy^3z^2)$, find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ at the point $(1, -1, 1)$. (06 Marks)
 - Show that $\vec{F} = (y + z)i + (z + x)j + (x + y)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. (07 Marks)
 - Prove that $\nabla^2(\log r) = \frac{1}{r^2}$ where $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$. (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

- 6 a. Find Laplace transform of $(2t + 3)^2$. **(05 Marks)**
 b. Find Laplace transform of $e^{2t} \cos 3t$. **(05 Marks)**
 c. Find $L\left\{\frac{\cos 2t - \cos 3t}{t}\right\}$. **(05 Marks)**
 d. Using Laplace transform, evaluate $\int_0^{\infty} e^{-2t} t \cos t \, dt$. **(05 Marks)**
- 7 a. Find inverse Laplace transform of $\frac{s}{s^2 + 4s + 13}$. **(06 Marks)**
 b. Find $L^{-1}\left\{\frac{1}{(s^2 + 3s + 2)(s + 3)}\right\}$. **(07 Marks)**
 c. Find $L^{-1}\left\{\log\left(\frac{s^2 + 1}{s^2 + s}\right)\right\}$. **(07 Marks)**
- 8 a. Solve the differential equation $y'' + 4y' + 3y = e^{-t}$ with $y(0) = 1$ and $y'(0) = 1$ by using Laplace transforms. **(10 Marks)**
 b. Solve by using Laplace transforms $\frac{dx}{dt} - 2x = \cos 2t$, $\frac{dy}{dt} + 2y = \sin 2t$ with $x = 1$, $y = 0$ at $t = 0$. **(10 Marks)**
